THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics

MATH2010F Advanced Calculus I
Homework 1
Due Date: 11:59pm, 6 June, 2025

1. If $v(t)$ is smooth for any t such that $a \leq t \leq b$. Show how the arc-length formula

$$S = \int_a^b \|v'(t)\| dt$$

can be derived by approximating the curve $v(t)$ by straight line segments and taking a limit.

2. Let $f : [a, b] \to \mathbb{R}$ and $g : [a, b] \to [a, b]$ be continuous. Using the $\varepsilon - \delta$ definition of continuity, show that

$$\lim_{x \to a} f(g(x)) = f(\lim_{x \to a} g(x)) = f(g(a)).$$

3. For a function f of one variable, construct a function such that

   (a) f is differentiable everywhere, and

   (b) f' is bounded everywhere, but

   (c) f' is not Riemann-integrable.